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FRACTIONAL D-BRANES AND THEIR GAUGE DUALS [1]

M. Bertolini [a], **P. Di Vecchia** [a], **M. Frau** [b], **A. Lerda** [c,b], **R. Marotta** [a], **I. Pesando**[b]

[a] *NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*

[b] *Dipartimento di Fisica Teorica, Università di Torino and I.N.F.N., Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy*

[c] *Dipartimento di Scienze e Tecnologie Avanzate Università del Piemonte Orientale, I-15100 Alessandria, Italy*

Abstract

We study the classical geometry associated to fractional D3-branes of type IIB string theory on $\mathbb{R}_4/\mathbf{Z}_2$ which provide the gravitational dual for $\mathcal{N}=2$ super Yang-Mills theory in four dimensions. As one can expect from the lack of conformal invariance on the gauge theory side, the gravitational background displays a repulson-like singularity. It turns out however, that such singularity can be excised by an enhançon mechanism. The complete knowledge of the classical supergravity solution allows us to identify the coupling constant of the dual gauge theory in terms of the string parameters and to find a logarithmic running that is governed precisely by the β function of the $\mathcal{N}=2$ super Yang-Mills theory.

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Contents

1 Introduction

After the seminal paper of 't Hooft on the large N expansion [1], many attempts to obtain a string theory out of QCD have been tried. Recently, a remarkable progress in this direction has been achieved with the Maldacena conjecture [2] which states that $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions is equivalent to type IIB string theory compactified on $AdS_5 \times S_5$. This provides the first concrete example of how a string theory can be extracted from a gauge theory. On the other hand, however, it was expected that a string model should emerge from a gauge field theory because of confinement, while the $\mathcal{N} = 4$ super Yang-Mills is in the Coulomb phase. Thus, a lot of effort has been recently devoted to extend the Maldacena conjecture and find new correspondences between string theories and non-conformal and less supersymmetric gauge theories. These attempts include the study of the renormalization group flow under a relevant perturbation in $\mathcal{N} = 4$ super Yang-Mills [3, 4, 5], the study of fractional branes on conifold singularities [6, 7, 8, 9], the study of the so-called $\mathcal{N} = 1^*$ theory [10], and the search for the geometry of the stable non-BPS D-branes [11].

The common feature found in all these different examples is that the classical geometric backgrounds have naked singularities of repulson type. In some of these cases however, an interesting phenomenon was discovered [12]: a massive probe moving in these backgrounds becomes tensionless before reaching the singularity. The geometric locus where this occurs is called enhançon. When this happens the supergravity approximation is not valid beyond the enhançon and one is forced to consider stringy effects which should change the description and eventually excise the singularity. This phenomenon has been analyzed also in Refs. [13, 14] for different configurations and in Ref. [15] for fractional D-branes on K3 orbifolds. In Ref. [8] however, it has been shown that the repulson singularity of fractional branes on conifolds can be removed already at the supergravity level by suitably deforming the conifold, thus obtaining a consistent gravitational dual that explains many features of the gauge theory. More recently, in Ref. [16] it has been shown that instead the resolution of the conifold singularity is not sufficient to regularize the gravitational background and also in this case an enhançon mechanism seems to be necessary.

In this paper we study the classical geometry generated by fractional D3-branes

of type IIB string theory on the orbifold $\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbf{Z_2}$. These are BPS configurations that are constrained to be at the orbifold fixed hyperplane and preserve eight supercharges. The dual gauge theory corresponding to a stack of M such fractional D3-branes is pure $\mathcal{N} = 2$ super Yang-Mills theory in four dimensions with gauge group $U(M)$. This is known to be non conformal and thus it is interesting to check whether the dual classical geometry displays a singular behavior. Some features of this solution indicating that this is indeed the case were already found in Refs. [6, 17]. In this paper we give the complete solution, with all physical quantities expressed in terms of the string parameters α' and g_s, and analyze its properties in some detail.

The first step for finding the exact solution is done using the boundary state formalism that allows to determine which supergravity fields are coupled to the brane and also their asymptotic behavior at large distance. Using this formalism it is possible to infer the complete world-volume action for the fractional D3-branes and thus to obtain the non homogeneous equations that the supergravity fields must satisfy. These equations can be explicitly solved and one can check that they describe configurations which satisfy the no-force condition as implied by the BPS bound. As expected the classical solution exhibit a naked singularity, which is in fact a repulson. Following the analysis done in Refs. [12, 13, 14], we see that the enhançon mechanism works also in our case. Hence the region of validity of the supergravity approximation does not include the singularity. The properties of our solution suggest a physical picture where the enhançon geometry is that of a ring like in Ref. [13, 14], instead of an hypersphere as in Ref. [12].

Exploiting the detailed knowledge of the solution of the fractional D3-brane and its world-volume action, we are able to find the metric of the moduli space of the dual gauge theory and determine from it the Yang-Mills coupling constant $g_{\rm YM}$ in terms of the string parameters. We find that $g_{\rm YM}$ is logarithmically running with a β-function that exactly matches the one of $\mathcal{N} = 2$ super Yang-Mills theory in four dimensions.

2 The geometry of fractional D3-branes

The action for type IIB supergravity in ten dimensions can be written (in the Einstein frame) as [1]

$$S_{\rm IIB} = \frac{1}{2\kappa_{10}^2}\Bigg\{\int d^{10}x\,\sqrt{-\det G}\;R - \frac{1}{2}\int\Big[d\phi\wedge{}^*d\phi + {\rm e}^{-\phi}H_{(3)}\wedge{}^*H_{(3)} + {\rm e}^{2\phi}\,F_{(1)}\wedge{}^*F_{(1)}$$
$$+\ {\rm e}^{\phi}\,\widetilde{F}_{(3)}\wedge{}^*\widetilde{F}_{(3)} + \frac{1}{2}\,\widetilde{F}_{(5)}\wedge{}^*\widetilde{F}_{(5)} + C_{(4)}\wedge H_{(3)}\wedge F_{(3)}\Big]\Bigg\} \qquad (1)$$

[1] Our conventions for curved indices and forms are the following: $\varepsilon^{0\ldots9} = +1$, signature $(-,+^9)$, $\omega_{(n)} = \frac{1}{n!}\,\omega_{\mu_1\ldots\mu_n}dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_n}$, and $*\omega_{(n)} = \frac{\sqrt{-\det G}}{n!\,(10-n)!}\,\varepsilon_{\nu_1\ldots\nu_{10-n}\mu_1\ldots\mu_n}\,\omega^{\mu_1\ldots\mu_n}dx^{\nu_1}\wedge\ldots\wedge dx^{\nu_{10-n}}$.

where

$$H_{(3)} = dB_{(2)} \quad , \quad F_{(1)} = dC_{(0)} \quad , \quad F_{(3)} = dC_{(2)} \quad , \quad F_{(5)} = dC_{(4)} \tag{2}$$

are respectively the field strengths corresponding to the NS-NS 2-form potential, and to the 0-form, the 2-form and the 4-form potentials of the R-R sector, and

$$\widetilde{F}_{(3)} = F_{(3)} - C_{(0)} \wedge H_{(3)} \quad , \quad \widetilde{F}_{(5)} = F_{(5)} - C_{(2)} \wedge H_{(3)} \quad . \tag{3}$$

Moreover, $\kappa_{10} = 8\,\pi^{7/2}\, g_s\, \alpha'^2$ where g_s is the string coupling constant, and the self-duality constraint ${}^*\widetilde{F}_{(5)} = \widetilde{F}_{(5)}$ has to be implemented on shell. The Dp-branes with p odd are solutions of the classical field equations that follow from the action (1), which are charged under the R-R $(p+1)$-form potentials and preserve sixteen supercharges. The D3-brane solution, in which only the metric and the 4-form potential $C_{(4)}$ are turned on, is particularly important because of the AdS/CFT correspondence [2].

Let us now consider type IIB supergravity on the orbifold

$$\mathbb{R}^{1,5} \times \mathbb{R}^4/\mathbf{Z_2} \tag{4}$$

where $\mathbf{Z_2}$ is the reflection parity that changes the sign to the four coordinates of $\mathbb{R}^4$, which we take to be x^6, x^7, x^8 and x^9. This is to be understood as the singular limit of the corresponding ALE manifold. The bulk action for this theory is still given by eq.(1), but with κ_{10} replaced by $\kappa_{\rm orb} = \sqrt{2}\,\kappa_{10} = (2\pi)^{7/2}\, g_s\, \alpha'^2$. In this case, besides the usual Dp-branes (bulk branes) which can freely move in the orbifolded directions, there are also *fractional* Dp-branes [18] which are instead constrained to stay at the orbifold fixed hyperplane $x^6 = x^7 = x^8 = x^9 = 0$. These fractional branes are the most elementary configurations of the theory, preserve eight supercharges and can be viewed as D$(p+2)$-branes wrapped on the (supersymmetric) vanishing 2-cycle of the orbifold.

In this paper we will consider in detail the fractional D3-brane. From the supergravity point of view, this is a configuration in which the dilaton ϕ and the axion $C_{(0)}$ are constant, while the metric, the 4-form $C_{(4)}$ and the two 2-forms $B_{(2)}$ and $C_{(2)}$ are non-trivial. More precisely, the latter fields, whose presence is a distinctive feature of the fractional branes, are

$$C_{(2)} = c\,\omega_2 \quad , \quad B_{(2)} = b\,\omega_2 \tag{5}$$

where ω_2 is the 2-form dual to the vanishing 2-cycle of the orbifold, and c and b are scalar fields living in $\mathbb{R}^{1,5}$.

The fact that these are the non-trivial fields for a fractional D3-brane has a natural interpretation from a string theory point of view. In fact, let us consider the vacuum energy Z between two fractional D-branes which is given by the one-loop open string amplitude

$$Z = \int_0^\infty \frac{ds}{s}\, \mathrm{Tr}_{\mathrm{NS-R}} \left[\left(\frac{1+(-1)^F}{2} \right) \left(\frac{1+g}{2} \right) \mathrm{e}^{-2\pi s (L_0 - a)} \right] \tag{6}$$

where $(-1)^F$ is the GSO parity, g is the orbifold $\mathbf{Z}_2$ parity, and the intercept is $a = 1/2$ in the NS sector and $a = 0$ in the R sector. By making the modular transformation $s \to 1/s$, one can translate the one-loop open string amplitude (6) into a tree-level closed string exchange diagram and, after factorization, one can obtain the boundary state $|B\rangle$ associated to the fractional brane [19, 20] (for a review of the boundary state formalism and its applications see, for example, [21]). The boundary state represents the source for the closed strings emitted by the brane and in this case it has four different components which correspond to the (usual) NS-NS and R-R untwisted sectors and to the NS-NS and R-R twisted sectors. By saturating the boundary state $|B\rangle$ with the massless closed string states of the various sectors, one can determine which are the fields that couple to the fractional brane. In particular, following the procedure found in [22] and reviewed in [21], one can find that in the untwisted sectors the fractional D3-brane emits only the graviton $h_{\mu\nu}$ [2] and the 4-form potential $C_{(4)}$. The couplings of these fields with the boundary state are explicitly given by [15]

$$\langle B|h\rangle = -\frac{T_3}{\sqrt{2}}\, h_\alpha{}^\alpha\, V_4 \quad , \quad \langle B|C_{(4)}\rangle = \frac{T_3}{\sqrt{2}\,\kappa_{\rm orb}}\, C_{0123}\, V_4 \tag{7}$$

where $T_p = \sqrt{\pi}\,(2\pi\sqrt{\alpha'})^{(3-p)}$ is the normalization of the boundary state, which is related to the brane tension in units of the gravitational coupling constant [22], V_4 is the (infinite) world-volume of the D3-brane, and the index α labels the longitudinal directions. By doing this same analysis in the twisted sectors, we find that the boundary state of the fractional D3-brane emits a massless scalar $\widetilde{b}$ from the NS-NS sector and a 4-form potential $A_{(4)}$ from the R-R sector. Of course these fields exist only at the orbifold fixed hyperplane $x^6 = x^7 = x^8 = x^9 = 0$, and thus their dynamics develops in the remaining six-dimensional space. The couplings of these fields with the boundary state turn out to be given by [15]

$$\langle B|\widetilde{b}\rangle = -\frac{T_3}{\sqrt{2}\,\kappa_{\rm orb}}\,\frac{1}{2\pi^2\alpha'}\,\widetilde{b}\,V_4 \quad , \quad \langle B|A_{(4)}\rangle = \frac{T_3}{\sqrt{2}\,\kappa_{\rm orb}}\,\frac{1}{2\pi^2\alpha'}\,A_{0123}\,V_4 \quad . \tag{8}$$

The twisted fields $\widetilde{b}$ and $A_{(4)}$ are related to the fields b and c of eq.(5). In fact, the scalar $\widetilde{b}$ represents the fluctuation part of b around the background value which is characteristic of the $\mathbf{Z}_2$ orbifold [23, 20]

$$b = \frac{1}{2}\,(4\pi^2\alpha') + \widetilde{b} \quad , \tag{9}$$

while the potential $A_{(4)}$ is dual (in the six dimensional sense) to the scalar c. To write down this duality relation in a correct way, let us observe that the field equation for $C_{(2)}$ that follows from the action (1) is

$$d\,{}^*dC_{(2)} = F_{(5)} \wedge H_{(3)} = d\left(C_{(4)} \wedge H_{(3)}\right) \quad , \tag{10}$$

[2] We recall that the graviton field and the metric are related by $G_{\mu\nu} = \eta_{\mu\nu} + 2\kappa_{\rm orb}\, h_{\mu\nu}$.

so that we can write

$$ {}^*dC_{(2)} - C_{(4)} \wedge H_{(3)} = dA_{(6)} \quad . \tag{11} $$

The 6-form $A_{(6)}$ is the dual (in the ten dimensional sense) to the R-R 2-form $C_{(2)}$. Let us now write this relation in the case of the wrapped brane, *i.e.* using eq.(5) and taking $A_{(6)} = A_{(4)} \wedge \omega_2$. Then, one can easily find that

$$ dA_{(4)} = - {}^{*_6}dc - C_{(4)} \wedge d\,b \tag{12} $$

where the Hodge dual $*_6$ is taken in the six-dimensional space where the twisted fields live.

From the explicit couplings (7) and (8), it is possible to infer the form of the world-volume action of a fractional D3-brane, namely

$$ \begin{aligned} S_{\rm boundary} \;\; = \;\; & -\frac{T_3}{\sqrt{2}\,\kappa_{\rm orb}} \int d^4x \sqrt{-\det G_{\alpha\beta}} \left(1 + \frac{1}{2\pi^2\alpha'}\,\widetilde{b}\right) \\ & + \frac{T_3}{\sqrt{2}\,\kappa_{\rm orb}} \int C_{(4)} \left(1 + \frac{1}{2\pi^2\alpha'}\,\widetilde{b}\right) + \frac{T_3}{\sqrt{2}\,\kappa_{\rm orb}} \frac{1}{2\pi^2\alpha'} \int A_{(4)} \quad . \end{aligned} \tag{13} $$

Of course, the boundary state calculations only determine the linear terms of the world-volume action, but the higher order terms can be found for example by imposing reparametrization invariance on the world-volume (first line of (13)) or by considering the WZ part of the action of a D5-brane wrapped on the (vanishing) 2-cycle in the presence of a non-trivial $B_{(2)}$ field (second line of (13)). The structure of the boundary action $S_{\rm boundary}$ is confirmed also by explicit calculations of closed string scattering amplitudes on a disk with appropriate boundary conditions [24].

As explained in Ref. [22], the boundary state formalism allows also to compute the asymptotic behavior at large distance of the various fields in the classical brane solution. For example, in our fractional D3-brane we find that the metric is

$$ ds^2 \simeq \left(1 - \frac{Q}{2\,r^4}\right) \eta_{\alpha\beta}\, dx^\alpha dx^\beta + \left(1 + \frac{Q}{2\,r^4}\right) \delta_{ij}\, dx^i dx^j \; + ... \tag{14} $$

where $\alpha, \beta = 0, ..., 3$; $i, j = 4, ..., 9$; $r = \sqrt{x^i x^j \delta_{ij}}$ and

$$ Q \equiv \frac{\kappa_{\rm orb}\, T_3}{2\sqrt{2}\,\pi^3} = 4\pi\, g_s\, \alpha'^2 \quad , \tag{15} $$

while the untwisted 4-form potential is

$$ C_{(4)} \simeq -\frac{Q}{r^4}\; dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 + ... \quad . \tag{16} $$

The asymptotic behavior of the twisted fields is instead given by

$$ \begin{aligned} \widetilde{b} \;\; & \simeq \;\; K \,\log(\rho/\epsilon) + ... \quad , & (17) \\ A_{(4)} \;\; & \simeq \;\; K \,\log(\rho/\epsilon)\; dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 + ... \quad , & (18) \end{aligned} $$

where $\rho = \sqrt{(x^4)^2 + (x^5)^2}$; ϵ is a regulator and

$$K \equiv \frac{\kappa_{\rm orb}\, T_3}{\sqrt{2}\,\pi}\;\frac{1}{2\pi^2\alpha'} = 4\pi\, g_s\, \alpha' \quad . \tag{19}$$

It is interesting to observe that while the untwisted fields depend on the radial coordinate r of the entire six-dimensional transverse space, the twisted fields which do not see the four orbifolded directions, depend only on the radial coordinate ρ of the remaining two-dimensional transverse space. This particular feature was also found in Ref. [25] in the case of the non-BPS D-branes in non-compact orbifolds, while the logarithmic asymptotic behavior of the twisted fields was already pointed out in Refs. [6, 26].

In the following we look for an exact solution of the field equations of type IIB supergravity with the asymptotic behavior described above. We start by writing the equations of motion for the dilaton ϕ and the axion $C_{(0)}$, which are

$$d^*d\phi = \mathrm{e}^{2\phi}\, dC_{(0)} \wedge {}^*dC_{(0)} + \frac{1}{2}\,\mathrm{e}^{\phi}\, \widetilde{F}_{(3)} \wedge {}^*\widetilde{F}_{(3)} - \frac{1}{2}\,\mathrm{e}^{-\phi}\, H_{(3)} \wedge {}^*H_{(3)} \quad , \tag{20}$$

and

$$d\left(\mathrm{e}^{2\phi}\, {}^*dC_{(0)}\right) = -\,\mathrm{e}^{\phi}\widetilde{F}_{(3)} \wedge {}^*H_{(3)} \quad . \tag{21}$$

As we discussed above, we are interested in a solution in which both the dilaton and the axion are constant, and the two 2-form potentials are as in eq.(5). To obtain this solution, it is convenient to introduce the combination [3]

$$G_{(3)} = F_{(3)} - \tau H_{(3)} \quad \text{with} \quad \tau = C_{(0)} + \mathrm{i}\,\mathrm{e}^{-\phi} \quad . \tag{22}$$

For constant dilaton and axion, eqs. (20) and (21) imply that

$$G_3 \wedge {}^*G_3 = 0 \tag{23}$$

which, using eq.(5), in turn implies that

$$d\gamma \wedge {}^{*_6}d\gamma \wedge \omega_2 \wedge \omega_2 = 0 \tag{24}$$

where we have defined the complex scalar

$$\gamma = c - \,\tau\, \widetilde{b} \tag{25}$$

and taken into account the anti-selfduality of ω_2. If $d\gamma \wedge {}^{*_6}d\gamma$ has components along x^4 and x^5, *i.e.* along the transverse directions orthogonal to the orbifold, then in order to satisfy eq.(24) we must require that

$$\partial_z\gamma\;\partial_{\bar{z}}\gamma = 0 \quad \text{where} \quad z = x^4 + \mathrm{i}\, x^5 \tag{26}$$

[3]Note that $G_{(3)}$ is not the $Sl(2,\mathbb{R})$ invariant 3-form that is usually used in the supergravity literature, but differs from the latter simply by a multiplicative factor.

which clearly can be satisfied by taking γ to be, for instance, an analytic function of z [6]. If we do this, the dilaton and the axion can be consistently taken to be constant, and, without any loss of generality, we set them to zero. With this choice, of course we have $\tau = \mathrm{i}$.

Let us now turn to the other field equations. To derive them, it is convenient to first insert the *Ansatz* (5) into the original action (1) and then use the fact that the integral of the product of two forms ω_2 over the four-dimensional orbifolded space is a constant that we choose so that the various fields in the bulk action have the canonical normalization, apart from the overall factor of $1/(2\kappa_{\rm orb}^2)$. Proceeding in this way, we obtain the following action

$$\begin{aligned} S'_{IIB} &= \frac{1}{2\kappa_{\rm orb}^2}\Bigg\{\int d^{10}x\,\sqrt{-\det G}\,R - \frac{1}{4}\int \widetilde{F}_{(5)} \wedge {}^*\widetilde{F}_{(5)} \\ &\qquad - \frac{1}{2}\int\Big[d\bar\gamma\wedge {}^{*6}d\gamma - \frac{\mathrm{i}}{2}C_{(4)}\wedge d\gamma\wedge d\bar\gamma\Big]_6\Bigg\} \end{aligned} \tag{27}$$

where the subindex 6 in the second line indicates that the integral is over the six-dimensional space orthogonal to the orbifolded directions. At this point we can write the field equations that follow from the total action $S = S'_{\rm IIB} + S_{\rm boundary}$. The equation for the 4-form potential $C_{(4)}$ is [4]

$$d\,{}^*\widetilde{F}_{(5)} + \frac{\mathrm{i}}{2}\,d\gamma\wedge d\bar\gamma\wedge\Omega_4 + \left(\frac{2\kappa_{\rm orb}T_3}{\sqrt{2}}\right)\Omega_2\wedge\Omega_4 = 0 \tag{28}$$

where we have defined

$$\begin{aligned} \Omega_4 &= \delta(x^6)\cdots\delta(x^9)\;dx^6\wedge\cdots\wedge dx^9 \quad , \\ \Omega_2 &= \delta(x^4)\,\delta(x^5)\;dx^4\wedge dx^5 \quad , \end{aligned} \tag{29}$$

while the equation for the complex scalar γ is

$$\Big(d\,{}^{*6}d\gamma + \mathrm{i}\,\widetilde{F}_{(5)}\wedge d\gamma\Big)\wedge\Omega_4 + \mathrm{i}\left(\frac{2\kappa_{\rm orb}T_3}{\sqrt{2}}\right)\frac{1}{2\pi^2\alpha'}dx^0\wedge\cdots\wedge dx^3\wedge\Omega_2\wedge\Omega_4 = 0 \;\; . \tag{30}$$

Finally, the field equations for the metric are

$$\widetilde{R}_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4\cdot 4!}(\widetilde{F}_{(5)})_{\mu\lambda_1\ldots\lambda_4}(\widetilde{F}_{(5)})_\nu{}^{\lambda_1\ldots\lambda_4} = L_{\mu\nu} \tag{31}$$

where

$$L_{\alpha\beta} = -\frac{L}{\sqrt{-\det G}}\,G_{\alpha\beta} \quad , \quad L_{ij} = \frac{L}{\sqrt{-\det G}}\,G_{ij} \quad , \tag{32}$$

[4] Note that, as usual, only the linear part of boundary action gives a non-trivial contribution to the field equations.

with

$$L = \left(\frac{1}{8} \sqrt{- \det G_6} \; \partial \gamma \cdot \partial \bar{\gamma} + \frac{\kappa_{\rm orb} T_3}{2\sqrt{2}} \sqrt{- \det G_{\alpha\beta}} \; \delta(x^4) \, \delta(x^5) \right) \delta(x^6) \cdots \delta(x^9) \quad . \tag{33}$$

We remark that in writing eq.(33) we have used the analyticity of γ, and have denoted by G_6 the metric in the six-dimensional space orthogonal to the orbifold.

We now solve eqs.(28), (30) and (31) by using a 3-brane-like Ansatz for the untwisted fields, namely

$$ds^2 = H^{-1/2} \, \eta_{\alpha\beta} \, dx^\alpha dx^\beta + H^{1/2} \, \delta_{ij} \, dx^i dx^j \quad , \tag{34}$$

$$\widetilde{F}_{(5)} = d \left(H^{-1} \, dx^0 \wedge \ldots \wedge dx^3 \right) + {}^* d \left(H^{-1} \, dx^0 \wedge \ldots \wedge dx^3 \right) \quad . \tag{35}$$

Inserting these expressions into eq.(30), we easily obtain

$$\delta^{ab} \, \partial_a \partial_b \, \gamma + \mathrm{i} \, \frac{2\kappa_{\rm orb} T_3}{\sqrt{2}} \, \frac{1}{2\pi^2 \alpha'} \, \delta(x^4) \, \delta(x^5) = 0 \tag{36}$$

where $a, b = 4, 5$, whose analytic solution is

$$\gamma = - \, \mathrm{i} \, K \, \log(z/\epsilon) \tag{37}$$

where K is defined in eq.(19) and $z = x^4 + \mathrm{i} \, x^5$. Taking the real and imaginary parts of γ, we get the twisted scalars

$$c = K \, \tan^{-1}(x^5/x^4) \quad , \tag{38}$$

$$\widetilde{b} = K \, \log(\rho/\epsilon) \quad . \tag{39}$$

It is interesting to see that the asymptotic behavior of $\widetilde{b}$ given in eq.(17) coincides with the complete solution (39). Furthermore, using the duality relation (12) and the *Ansatz* (34)-(35), we can obtain the classical profile of the twisted R-R potential $A_{(4)}$ appearing in the boundary action of the fractional D3-brane. In fact, we have

$$A_{(4)} = K \, \log(\rho/\epsilon) \, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \quad . \tag{40}$$

Again the asymptotic form (18) obtained from the boundary state coincides with the full solution (40).

Let us now find the equation that determines the warp factor H. Inserting the *Ansatz* (34)-(35) into eq.(28), we get

$$\delta^{ij} \, \partial_i \partial_j H + |\partial_z \gamma|^2 \, \delta(x^6) \ldots \delta(x^9) + \frac{2\kappa_{\rm orb} T_3}{\sqrt{2}} \, \delta(x^4) \ldots \delta(x^9) = 0 \quad . \tag{41}$$

The last contribution is the standard source term that is present also for the usual bulk D3-branes, while the second contribution is a peculiar feature of the fractional D3-branes and represents the fact that, in this case, the non-trivial flux of $G_{(3)}$ is a

source for the untwisted fields [17, 27]. The final consistency check is to show that eq. (41) follows also from the Einstein equation (31). This is indeed what happens; in fact, using our *Ansatz*, it is possible to show that the left-hand side of eq.(31) becomes

$$\widetilde{R}_{\alpha\beta} = \frac{\delta^{ij}\partial_i\partial_j H}{4\,H^2}\,\eta_{\alpha\beta} \quad , \quad \widetilde{R}_{ij} = -\frac{\delta^{lk}\partial_l\partial_k H}{4\,H}\,\delta_{ij} \quad , \tag{42}$$

while the right-hand side becomes

$$\begin{aligned} L_{\alpha\beta} &= -\frac{1}{4\,H^2}\left(|\partial_z\gamma|^2\,\delta(x^6)\ldots\delta(x^9) + \frac{2\kappa_{\rm orb}T_3}{\sqrt{2}}\,\delta(x^4)\ldots\delta(x^9)\right)\eta_{\alpha\beta} \quad , \\ L_{ij} &= \frac{1}{4\,H}\left(|\partial_z\gamma|^2\,\delta(x^6)\ldots\delta(x^9) + \frac{2\kappa_{\rm orb}T_3}{\sqrt{2}}\,\delta(x^4)\ldots\delta(x^9)\right)\delta_{ij} \quad . \end{aligned} \tag{43}$$

Hence, also the Einstein equation (31) implies eq.(41). Using standard techniques, it is possible to find the explicit solution of this equation, and H reads

$$H = 1 + \frac{Q}{r^4} + \frac{2K^2}{r^4}\left[\log\left(\frac{r^4}{\epsilon^2(r^2-\rho^2)}\right) - 1 + \frac{\rho^2}{r^2-\rho^2}\right] \quad . \tag{44}$$

This expression is clearly in agreement with the results (14) and (16) obtained from the boundary state.

Having the explicit form of the solution, we can now analyze its properties. First of all, we can check that it respects the no-force condition, as it should be because of its BPS properties. To see this, we can substitute our classical solution into the boundary action (13) and find

$$\begin{aligned} S_{\rm boundary} &= -\frac{T_3\,V_4}{\sqrt{2}\,\kappa_{\rm orb}}\left\{H^{-1}\left[1 + \frac{K}{2\pi^2\alpha'}\,\log(\rho/\epsilon)\right]\right. \\ &\quad \left. -\,(H^{-1}-1)\left[1 + \frac{K}{2\pi^2\alpha'}\,\log(\rho/\epsilon)\right] - \frac{K}{2\pi^2\alpha'}\,\log(\rho/\epsilon)\right\} \\ &= -\frac{T_3\,V_4}{\sqrt{2}\,\kappa_{\rm orb}} \quad . \end{aligned} \tag{45}$$

The fact that all position dependent terms exactly cancel leaving a constant result is a check on the no-force condition to all orders; therefore, one can safely form a stack of M fractional D3-branes by simply piling them on top of each other. In this case, the solution for such a configuration has still the same form as before, but with $Q \to M\,Q$ and $K \to M\,K$.

On the other hand, a closer look at the behavior of the function H in eq.(44) shows that the metric of the fractional D3-branes has a naked singularity. As we discussed in the introduction, this fact is a feature that is shared also by other configurations which are dual to non-conformal gauge theories [12, 7, 13, 14, 16, 11, 15], and indeed possess a naked singularity at some $r = r_0$. Actually, the structure of such singularity is that of a *repulson* because in its vicinity the gravitational

force, which is related to the gradient of the temporal component of the metric tensor, becomes repulsive. Thus, there exists a region of anti-gravity and a distance $r = r_e > r_0$ where the gravitational force vanishes. A study of the shape of G_{00} indicates that the singularity is not a point in the transverse six dimensional space but rather a two-dimensional surface. In fact, the repulson is located near $x^6 = x^7 = x^8 = x^9 = 0$ and extends along the non-orbifolded transverse directions x^4 and x^5. Note that the breaking of the spherical symmetry in the six-dimensional transverse space is not surprising since the starting vacuum geometry, eq.(4), already breaks it. Moreover, a simple numerical analysis reveals that the singularity does not cover the full x^4, x^5 plane; in fact the temporal component of the metric tensor ceases to be singular at some value ρ_0 and smoothly goes to zero for bigger values of ρ, signaling the possible appearance of an horizon. Clearly, a more detailed analytical study of the classical geometry produced by a fractional D3-brane and of its singularity is needed. Here we have just mentioned the most relevant features which are useful and sufficient for the discussion of the following section.

3 The enhançon and the dual gauge theory

The discussion of the previous section and the presence in our solution of a geometrical locus where the gravitational force vanishes, clearly indicates the possibility that an enhançon mechanism may take place [12] [5]. This would excise the repulson from the metric, thus yielding a singularity-free solution.

In order to see whether this really happens, we apply the same methods of Ref. [12] and study the (slow) motion of a probe brane in the geometry generated by M fractional D3-branes (for a review on this approach see, for example, [28]). This can be done by inserting the classical solution into the world-volume action (13) and expanding it in powers of the velocity of the probe D3-brane in the two transverse directions x^4 and x^5. Doing this, we find that the position dependent terms cancel exactly because of supersymmetry, as we have already seen in the previous section, while the terms quadratic in the velocity of the probe survive and allow to define a non-trivial two-dimensional metric on moduli space. More precisely, from the DBI part of boundary action, we get

$$\frac{1}{2}\frac{T_3}{\sqrt{2}\kappa_{\rm orb}}\int d^4x \left(\frac{\partial x^a}{\partial x^0}\right)^2\left(1+\frac{\widetilde{b}}{2\pi^2\alpha'}\right) \tag{46}$$

where the index a takes values 4 and 5. We now express the various constants of (46) in terms of the string parameters, use eq.(39) and identify the coordinates $x^a = 2\pi\alpha'\Phi^a$ with the Higgs fields, so that the kinetic term for the scalars can be

[5]Notice that in [12], the enhançon locus coincides with the locus where the gravitational force vanishes, but this may not be necessarily the case for more general configurations. The "physical" enhançon occurs, by definition, where the probe brane becomes tensionless.

rewritten as follows

$$-\frac{1}{2}\int d^4x\ \partial_\mu\Phi^a\partial^\mu\Phi^b\, g_{ab} \tag{47}$$

where the metric in moduli space is

$$g_{ab} = \frac{1}{8\pi g_s}\left(1+\frac{M\,K}{2\pi^2\alpha'}\log(\rho/\epsilon)\right)\delta_{ab} \quad . \tag{48}$$

It is easy to see that this metric vanishes when ρ reaches the following value

$$\frac{\rho_e}{\epsilon} = \mathrm{e}^{-\pi/(2Mg_s)} \quad . \tag{49}$$

This means that at $\rho = \rho_e$ the probe fractional brane becomes tensionless; thus for $\rho < \rho_e$ the supergravity solution looses its meaning because there the probe gets a negative tension. This fact can be interpreted also as the signal that new degrees of freedom are becoming massless below ρ_e and that they have to be suitably taken into account with a fully stringy description. A phenomenon similar to the one originally discussed in [12] is at work here: the true microscopic configuration is not given by a stack of coincident branes but rather by an hypersurface (defined by eq.(49)) on which the branes are smeared. However, differently from [12], our enhançon is not an (hyper)sphere in the transverse space, but rather a ring depending on the two coordinates, x^4 and x^5 through ρ. Another important point to notice is that at the enhançon, the fluctuation part $\widetilde{b}$ of the twisted scalar field b exactly cancels its background value which, in unit of the string length $2\pi\sqrt{\alpha'}$, is $1/2$ (see eqs.(9) and (39)).

A more detailed characterization of the structure of the regularized classical solution deserves further study. Nevertheless, what we have found here is already enough to get interesting information about the dual field theory. We recall that in the case of M fractional D3-branes, the world-volume gauge theory is pure $\mathcal{N}=2$ super Yang-Mills in four dimensions with gauge group $U(M)$. This can be simply understood by analyzing the massless spectrum of the open strings attached to the fractional D3-branes. Notice that no hypermultiplets are present since the corresponding moduli would be related to displacements of the branes from the orbifold fixed point, which are not possible for fractional branes [6].

Remembering that the Higgs fields Φ^a are the two scalars of the $\mathcal{N}=2$ vector multiplet, from the action (47) we can read that the Yang-Mills coupling constant is given by

$$g^2_{\rm YM}(\mu) = (g^0_{\rm YM})^2\left(1+\frac{M\,(g^0_{\rm YM})^2}{4\pi^2}\log\mu\right)^{-1} \tag{50}$$

where

$$(g^0_{\rm YM})^2 \equiv g^2_{\rm YM}(\mu=1) = 8\pi g_s \quad , \quad \mu \equiv \frac{\rho}{\epsilon} \quad . \tag{51}$$

[6] This is to be contrasted with the case of N bulk branes where the gauge group is $U(N)\times U(N)$ and one expects also hypermultiplets to be present [29].

Eq.(50) defines the running of the YM coupling constant with the variation of the scale μ. Notice that from this point of view, the enhançon locus (49) defines the scale at which $g_{\rm YM}$ diverges. Remembering that $\rho \to \infty$ corresponds to the ultraviolet limit in the dual field theory, we see that the coupling constant (50) describes an asymptotically free gauge theory! Finally, by computing the β-function, we find

$$\beta \equiv \mu \frac{\partial}{\partial \mu} g_{\rm YM}(\mu) = -\frac{g^3_{\rm YM}(\mu)}{8\pi^2} M \tag{52}$$

which is precisely the β-function of the pure $\mathcal{N} = 2$ super Yang-Mills theory (modulo instanton corrections). It would be interesting to investigate the relation between our results and those of Ref. [30] where the $\mathcal{N} = 2$ gauge theories are obtained from M5-branes of 11-dimensional supergravity wrapped on Riemann surfaces.

It is also worth pointing out that the R-R twisted scalar c of eq.(38) is directly related to the θ-angle of the YM theory. In fact, by introducing a gauge field F in the world-volume action of the probe D3-brane and expanding the WZ part in powers of F, we can read from the coefficient in front of the $\mathrm{Tr}(F \wedge F)$ term that

$$c = 2\pi\, \alpha'\, g_s\, \theta_{YM} \quad . \tag{53}$$

As a consequence, the complex scalar $\bar{\gamma} = c + \mathrm{i}\, b$ of the supergravity solution can be nicely written as

$$\bar{\gamma} = (2\pi\sqrt{\alpha'})^2\, g_s\, \tau \tag{54}$$

where τ is the standard combination of the YM coupling constant and θ-angle

$$\tau = \frac{\theta_{YM}}{2\pi} + \mathrm{i}\, \frac{4\pi}{g^2_{YM}} \quad . \tag{55}$$

We conclude by observing that it is straightforward to extend our analysis to the case in which there are N bulk D3-branes besides the M fractional ones considered so far. The only change in the classical solution corresponding to this configuration occurs in the function H of eq.(44) in which the parameter Q must be replaced by $(2N + M)\, Q$, while K must be replaced by $M\, K$ as before. Furthermore, the enhançon locus gets changed to

$$\frac{\rho_e}{\epsilon} = \mathrm{e}^{-\pi(1+2n)/(2Mg_s)} \tag{56}$$

where $n = N/M$.

Our results show that is possible to obtain precise non-perturbative information on a gauge theory using the dual classical geometry provided by D-branes, even in cases different from those of the AdS/CFT correspondence. This fact hints the possibility that the gauge/gravity duality has an even deeper meaning than expected.

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